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WARTIME REPORT

JET-BOUNDARY CORRECTIONS TO THE DOWNWASH BEHIND POWERED

MODELS IN RECTANGULAR WIND TUNNELS WITH NUMERICAL

VALUES FOR 7- BY 10-FOOT CLOSED WIND TUNNELS

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ADVANCE RESTRICTED REPORT

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SUMMARY

Methods are presented for determining the jet-boundary corrections to the downwash behind models in rectangular wind tunnels. The methods take into account the tunnel dimensions, the type of jet, the span loading of the model, the geometric position of the wing and of the tail in the tunnel, the displacement of the wing wake, and the effect of the slipstream. A correction to the lift of the wing due to the curvature of the streamlines was determined from the downwash correction calculations and is included in the appendix. Numerical values of the downwash correction factors for 7- by 10-foot closed wind tunnels are presented in the form of graphs.

INTRODUCTION

The influence of the jet boundaries upon the down-wash at the wing and behind the wing has been rather extensively investigated from theoretical considerations and the theory has been roughly checked by experimental data. The general method of determining the correction factors to the downwash behind a wing in a rectangular tunnel was first given by Glauert and Hartshorn (reference 1). Although these authors gave the formula for an exact solution for the induced upwash velocity due to any given image of the vortex system assumed to replace the wing, they summed up the effect of all the images by the use of an approximate formula, valid only for models that are very small relative to the size of the wind tunnel. The experimental checks presented were also for relatively small models. The calculation methods of reference 1 were further developed and refined in references 2 and 3.

The factors usually considered in downwash-correction calculations are the tunnel dimensions, the type of jet, and the geometric position of the wing and tail in the tunnel. The wing is usually replaced by a simple horse-shoe vortex with an effective span approximately equal to the theoretical distance between the completely rolled-up trailing vortices.

The additional factors considered in the present investigation are the effect of the displacement of the vortex sheet, the effect of nonuniform span loading, and the influence of the slipstream. The methods necessary to determine the effect of all the various factors considered upon the corrections to the downwash angle and wake displacement are presented and discussed. Corresponding corrections to the measured pitching moments, the elevator hinge moments, and the elevator free-floating angle are presented. A correction to the lift of the wing due to the curvature of the streamlines was determined by use of the downwash correction factors and is given in appendix A. Theoretical values of the various correction factors for 7- by 10-foot closed wind tunnels are also presented.

THEORY

General Solution

The jet boundaries impose certain restrictions upon the air flow around a model. The known conditions to be satisfied are zero normal velocity at the boundaries for closed-type wind tunnels and constant pressure at the boundaries for open-type wind tunnels. It has been shown (reference 1) that the boundary conditions may be satis-fied by replacing the boundaries with a doubly infinite pattern of images of the model vortex system.

The image arrangement for a closed rectangular wind tunnel is illustrated in figures 1 and 2. (See reference 3 for the image arrangement for open-type tunnels.) In figure 1, a three-dimensional drawing, for simplicity the wing is shown on the center line of the wind tunnel. In figure 2, a three-view drawing, the wing and the tail are shown located off-center, as in the general case. The axes used are indicated in the figures and the terminology used in this paper is given in appendix B. The wing is

replaced by a simple horseshoe vortex in the figures and in most of the following calculations. Any actual non-uniform span lift distribution may be built up of several simple horseshoe vortices of various spans and strengths.

The problem of determining the numerical values of the vertical velocity w induced by the jet boundaries may be solved by calculating the vertical velocity due to each separate image and then summing up the effects of all the images. Inasmuch as the effectiveness of the images decreases rather rapidly as their distance from the actual wing increases, the effect of only a few of the more important images need be calculated. This method is necessary because no simple, exact, convergent series representing the effects of the whole doubly infinite system of images has been found. In reference 1 a fairly simple series is presented that is valid only for the case when the tail length and the vortex semispan distances are approximately equal and are quite small compared with the distance to the first set of images. For larger models (relative to the tunnel) the errors involved may become quite large, as was shown in reference 2.

The total vertical velocity w_{total} induced at the tail position $x,0,0$ by a simple positive image horseshoe vortex (positive image indicates an image similar to model vortex) located with the center section of the lifting line at $0,y,z$ is given by

$$w_{total} = \frac{\Gamma}{4\pi} \left\{ \left[\frac{y - s}{(y - s)^2 + z^2} - \frac{y + s}{(y + s)^2 + z^2} \right] \right.$$

$$+ \left[\frac{x(y - s)}{\sqrt{(y - s)^2 + x^2 + z^2}} \left(\frac{1}{x^2 + z^2} + \frac{1}{(y - s)^2 + z^2} \right) \right]$$

$$\left. - \left[\frac{x(y + s)}{\sqrt{(y + s)^2 + x^2 + z^2}} \left(\frac{1}{x^2 + z^2} + \frac{1}{(y + s)^2 + z^2} \right) \right] \right\} \quad (1)$$

where

4

Γ   circulation strength of horseshoe vortex

x   distance from lifting line of image to tail position,
      parallel to X axis

y   distance to center section of image lifting line,
      parallel to Y axis

z   distance to image lifting line, parallel to Z axis

s   semispan of simple horseshoe vortex

This formula is identical with that given in reference 1
except for signs, since downwash was considered positive
in reference 1 and in this report upwash is considered
positive. The total boundary-induced upwash velocity
w_{total} is given in equation (1) for a single image and
must be summed up for all images in order to determine
the complete downwash correction.

The upwash velocity at the center section of the
lifting line of the wing $w_{c.s.}$ is given by the first
set of terms of equation (1), that is, by the terms that
are independent of x. The other two sets of terms give
the additional upwash at the center section of the tail
$w_{a_{c,s.}}$, that is, the increase in upwash velocity at the
tail over that at the center section of the wing. In
order to calculate the correction to the measured pitch-
ing moments, the additional upwash with respect to the
average wing upwash should be used rather than the addi-
tional upwash with respect to the center-section wing
upwash. This additional upwash with respect to the aver-
age upwash across the wing is found from the total upwash
w_{total} and the average wing upwash w_w as follows:

$$w_{a_w} = w_{total} - w_w \qquad (2)$$

It will later be shown that this expression for w_{a_w} may
be used only for the power-off case and must be modified
for the power-on case.

Several calculations showed no appreciable difference
between the correction for the center of the tail and the

average correction for the entire tail for representative
models in 7- by 10-foot closed wind tunnels; hence, for
simplicity, the upwash velocity will be calculated only
for the center section of the tail. Calculations for a
7- by 20-foot tunnel, however, indicate that an appreci-
able difference between the upwash at the center section
and the average upwash across the tail might be expected
for some model-tunnel arrangements.

In general, it is necessary to calculate the value
of w_{total} and w_w due to each image vortex and then to
sum up the effects of all the image vortices. In practice
the calculations are considerably simplified because
w_{total} may be calculated as the sum of $w_{c.s.}$ and
$w_{a_{c.s.}}$; and $w_{c.s.}$, being independent of x, need be cal-
culated only once for all values of x. Simple summation
formulas (see equation (20) of this paper) may be used to
calculate the values of $w_{c.s.}$. The usual average wing
correction w_w is generally already available or can be
determined from reference 3. Also, as previously men-
tioned, the calculations of $w_{a_{c.s.}}$ (equation (21)) need
be made for only a few of the more important near-image
vortices. It is seldom necessary to compute the effect
of an image farther than five images away; the relation
between the height and the width of the tunnel, however,
determines how many images must be considered.

In order to determine which images are important,
the upwash velocity due to several images was calculated
for a point 3 feet behind the lifting line of a 3-foot
semispan horseshoe vortex at the center of a 7- by 10-
foot closed wind tunnel. The calculation was made to an
accuracy of approximately one-tenth of 1 percent of the
final summation value (five decimal places). Inasmuch as
a case with the wing on the center line of the tunnel was
selected, only zero and positive values of n and m
(see figs. 1 and 2) had to be calculated because the sign
of the integers does not affect the absolute magnitude of
the upwash velocity and the upwash velocity for the nega-
tive values of n or m is therefore equal to the up-
wash velocity for positive values of n or m.

The value of the upwash velocity, converted to the
nondimensional correction factor $\delta_{a_{c.s.}}$, is given in
table I for each of the images where

$$\delta = \frac{C}{4s} \frac{w}{\Gamma} \tag{3}$$

since

$$\Gamma = \frac{V_o S C_L}{4s}$$

The symbols used in these equations, which were not previously defined, are

C   tunnel area

V_o free-stream velocity

S   wing area

C_L lift coefficient

and the angular correction is determined from the correction factor by

$$\Delta\epsilon = \frac{w}{V_o} = \delta \frac{S}{C} C_L \tag{4}$$

where $\Delta\epsilon$ is small and is assumed equal to its tangent. The symbols δ and w may be subscripted as indicated in appendix B to apply to any component of boundary-induced upwash.

The sign of the upwash correction factors for values of upwash correction factors less than 0.00001 is given in table I to show that the summation is at least reasonably accurate, since approximately as many positive zeros as negative zeros are present. An additional correction due to the neglected images could be determined by the approximate formula of reference 1 but was not considered necessary because, by means of a careful selection of the important images, practically the same summation was obtained with a few images as with the entire pattern of 15 images. The important images selected for a 7- by 10-foot closed wind tunnel are indicated in table I. The summations obtained by the use of both the pattern of 15 images and the pattern of important images are also given in table I.

Modifications to General Solution

Span loading.- The span of the simple horseshoe
vortex, which is assumed to represent the model for pur-
poses of calculation, is usually taken to be approximately
the distance between the tip vortices of the completely
rolled-up vortex sheet. The complete rolling-up process
occurs quite slowly, as mentioned in reference 4, and
probably would seldom be accomplished in ordinary wind-
tunnel operation. Thus, the actual loading along the
wing span should be used for the accurate determination
of the jet-boundary corrections.

The average boundary-induced upwash velocity at the
lifting line, as calculated for 7- by 10-foot closed wind
tunnels by the methods of reference 3, usually has a
numerical value lower than the value calculated by use
of the actual span load distribution because the upwash
velocity is averaged across the assumed effective span
rather than across the actual span. An empirical rela-
tion between the actual and the effective model span was
determined in order effectively to average the upwash
velocity across the actual model span. Use of this empir-
ical effective span gives numerical values of w_w in
good agreement with the values obtained by use of the
actual span loading. The effective span for plain wings
in a 7- by 10-foot closed tunnel should be about 0.9 times
the actual span and for partial-span flaps should be about
equal to the actual flap span.

In order to account for the marked changes in span
loading due to partial-span flaps, the correction is
broken into two parts; that is,

$$\Delta \epsilon = \left[(\delta C_L)_w + (\delta C_L)_f \right] \frac{S}{C} = \left[(\delta C_L)_{w+f} \right] \frac{S}{C} \qquad (5)$$

where

$(\delta C_L)_w$ product of correction factor due to wing and
            increment of lift coefficient due to wing

$(\delta C_L)_f$ product of correction factor due to flap and
            increment of lift coefficient due to flap

This addition of the effects of the wing and the flap will be assumed to have been made in the following formulas; however, only the product $(\delta C_L)_{w+f}$ will be indicated. In many cases the correction factor δ will be very nearly equal for the wing and for the flap, and a single value may therefore be used with satisfactory accuracy.

Wake displacement.- An additional correction to the downwash is necessary because of the displacement of the wake or slipstream in the tunnel. The free-air displacement of the wake or slipstream center line is usually decreased by the boundary-induced upwash in a closed tunnel. As the stability and control characteristics of airplanes often depend critically upon the position of the wake or slipstream with respect to the tail surfaces, this boundary-induced displacement must be determined. The displacement z' of the wake or slipstream at a position x in free air is determined as

$$z' = \int_{T.E.}^{x} \tan \epsilon \; dx$$

where ϵ is the angle of inclination of the wake or slipstream at each point between the trailing edge of the wing or propeller disk and x. Because the angle ϵ will be different in free air and in the tunnel, the boundary-induced displacement is, to a first approximation,

$$\Delta z' = \int_{T.E.}^{x} \tan (\epsilon + \Delta \epsilon) \; dx - \int_{T.E.}^{x} \tan \epsilon \; dx$$

where $\epsilon + \Delta \epsilon$ is the measured inclination at each point on the wake or slipstream center line and $\Delta \epsilon$ is the corrected angle at that point. Unless ϵ is very large it will usually be satisfactory to determine $\Delta z'$ as

$$\Delta z' = \int_{T.E.}^{x} \Delta \epsilon \; dx \tag{6}$$

The correction to the displacement of the wake center line at each point x may also be considered to be the correction to the displacement of all points having an ordinate equal to x except for the points lying in the slipstream. A similar correction to the slipstream displacement must also be applied. In general, the angle $\Delta\epsilon$ will be different for the wake and for the slipstream, and the resulting displacement will be different.

The computed boundary-induced displacement at the tail for a typical 1/5-scale powered model in a 7- by 10-foot closed tunnel amounts to about $0.7\ C_L$ inch, or $3.5\ C_L$ inches on the full-scale airplane. Near maximum lift the correction thus becomes fairly large.

The correction may be easily applied only to downwash measurements. The measured pitching moments must also be corrected, however, because the displacement of the wake or slipstream may be large or the tail may be so critically located that a small change in location would change the tail effectiveness a great deal. This additional pitching-moment correction can be applied only by calculating the additional change in angle $\Delta\epsilon_{z'}$, occurring in free air (from the charts of reference 5), due to the change in vertical location $\Delta z'$ of the tail with respect to the wake. For a powered model, actual downwash measurement behind the model, or a similar model, must be used to estimate $\Delta\epsilon_{z'}$.

Distortion and rolling-up of vortex sheet.- The wing is represented by a series of simple horseshoe vortices extending uniformly downstream to infinity. The actual vortex sheet of the wing is known to be displaced vertically downward as well as rolled up after it leaves the trailing edge of the wing. Some modifications to the corrections must be made to account for the deviation of the actual vortex sheet from the assumed vortex sheet.

In the determination of the downwash behind a wing the distortion of the vortex sheet may usually be accounted for by simply considering the entire vortex sheet to be displaced vertically by an amount equal to the displacement at the tail position of the center section of the actual distorted vortex sheet. The rolling-up of the vortex sheet may generally be neglected (reference 4). Because this modification is usually sufficiently accurate

to be used in determining the first-order effect, the actual downwash behind a wing, it would appear to be sufficiently accurate to be used in determining the effect of the jet boundaries upon the downwash.

In the determination of jet-boundary corrections, the problem is somewhat different from the problem of determining the actual downwash behind a wing, because the point for which the induced velocity is being computed is at a great distance from the image vortices. For this case the tip vortices are of greater importance and, as the displacement of the tip vortices is very slight (reference 4), a somewhat smaller displacement of the vortex sheet than that indicated by the center-section displacement should be used.

The center-section displacement may be determined from reference 5. A very approximate calculation is, however, sufficiently accurate for this modification. The angle of inclination of the center of the wing wake for a lift coefficient of unity is approximately 0.1 radian for normal aspect ratios and taper ratios and for wing flaps with a span ratio greater than 0.6 the wing span. For very short partial-span flaps, the wake angle is approximately 0.2 radian.

The problem is now merely the determination of an effective height d, the distance the lifting line must be assumed to be above the center line of the tunnel to account for the vortex-sheet displacement. This effective height d then replaces the actual height in the calculations of the jet-boundary corrections.

If an effective displacement of about one-half that at the center section is assumed to give reasonable accuracy in determining the jet-boundary corrections, the effective height to be used will be:

$$d = d_g - \Delta d \tag{7}$$

where

$$\Delta d = 0.05 \ C_L \ x$$

or, for flap spans of less than 0.6 the wing span,

$$\Delta d = 0.1 \ C_L \ x$$

and d_g is the geometric height of the origin of the tip vortices above the tunnel center line. The origin of the tip vortices may be assumed to be located at the wing trailing edge of the 0.9 semispan station.

Effect of slipstream.- The jet-boundary corrections are altered rather markedly by the slipstream, especially when the slipstream velocity ratio is large. The slipstream changes the span load distribution, changes the distortion of the vortex sheet, and adds a stream function (that of the slipstream itself) to the conditions that must be met at the jet boundaries. The main effect of the slipstream is, however, to decrease the correction to the downwash angle inasmuch as a vector addition rather than a scalar addition of the jet-boundary induced upwash velocity with the slipstream velocity is required for the case where the point in question (the tail) lies inside the slipstream. The effect of the vector addition will be considered when the angular corrections are determined.

The problem of determining the boundary-induced upwash velocity for a model with a slipstream is essentially the problem of determining the upwash velocity due to a doubly infinite pattern of the image models, each with its slipstream. No adequate theoretical method of computing the increase in downwash behind an airplane due to a slipstream is available. At the rather large distance (in the Y and Z directions) from each image that it is desired to calculate the boundary-induced upwash velocity, the main effect of power probably is the upwash velocity caused by the jet boundaries of the inclined slipstreams; that is, when the propeller is inclined to the air stream, the slipstream will have a component of flow (the free-stream flow) normal to it. This component of original flow across the slipstream will induce an upwash velocity inside and outside the slipstream. The induced upwash velocity outside the slipstream may be calculated for each image by an extension of the methods presented in reference 6. It is necessary to assume a slipstream with two-dimensional characteristics for the calculations. The final formula, including the summation factor for the 7- by 10-foot closed wind tunnel, is as follows:

$$
w_s = \frac{q_s/q_0 - 1}{q_s/q_0 + 1} \frac{V_0 \sin 2\epsilon_s}{2} \sum \sum (-1)^m \frac{\cos 2\theta}{r^2}
$$

$$
= \frac{q_s/q_0 - 1}{q_s/q_0 + 1} V_0 \epsilon_s (0.044) \tag{8}
$$

where

r,θ   position of a point with respect to center line of
      a two-dimensional slipstream

q_s dynamic pressure in slipstream

q_o free-stream dynamic pressure

ϵ_s inclination of slipstream, radians

Numerical values of the increment of jet-boundary
induced upwash velocity w_s due to the inclined slip-
stream calculated from equation (8) agree very well with
the values of the increment calculated by the simple
method of using the increment of lift due to the propel-
ler and the correction factors involved for power-off
conditions, at least for conventional single-engine air-
plane models. Thus, for simplicity, it is suggested that
the total power-on lift coefficient and the power-off
correction factor be used to calculate the total boundary-
induced upwash.

CORRECTIONS

Downwash-angle correction.- The correction to the
downwash angle is obtained from the total upwash velocity
and the local velocity at the particular point. The cor-
rection is given in the following equation (for small
angles) rearranged to simplify the calculations:

$$\Delta\epsilon_d = \frac{w_{total}}{V} = \frac{1}{V/V_o} \times \frac{w_{total}}{V_o} \tag{9}$$

If the point for which the correction is to be determined
is inside the slipstream, the velocity V becomes V_s
and, if outside the slipstream, V_o. The correction given
in equation (9) transferred to correction-factor form and
converted to degrees is

$$\Delta\epsilon_d = \frac{1}{\sqrt{q/q_o}} \left(\delta_{total} \, C_{L_p}\right)_{w+f} \frac{S}{C} (57.3) \tag{10}$$

since

$$\frac{1}{V/V_0} = \frac{1}{\sqrt{q/q_0}}$$

and C_{L_p} is the power-on lift coefficient,

The ratio q/q_0 is determined, for the point in question, from the pitot head used to measure the angle of downwash. The factor δ_{total} is determined for the particular point in question (located at $x = x$ and $z = d_t$) for a simple horseshoe vortex with a span equal to the effective span (for the wing and the flap). If the point for which the downwash-angle correction is desired does not lie in the plane of symmetry, the value of the correction factor for the point in the plane of symmetry may be used for most model-tunnel arrangements with satisfactory accuracy. Values of S, C_{L_p}, and δ_{total} depend upon the model used in the tests and values of δ_{total}, C_{L_p}, and q/q_0 vary with model attitude.

Correction for displacement of wake or slipstream.- The correction to the displacement of the wake or the slipstream may be determined from equations (6) and (10) in correction-factor form as

$$\Delta z' = \int_{T.E.}^{x} \frac{1}{\sqrt{q/q_0}} (\delta_{total} C_{L_p})_{w+f} \frac{S}{C} dx \qquad (11)$$

where q/q_0 and δ_{total} are functions of x. This displacement correction is fairly large and is important for conventional powered models.

Pitching-moment correction.- The angular correction to be used in determining the correction to the pitching moments is the difference in the boundary-induced air-flow angle at the tail and the average boundary-induced air-flow angle over the wings:

$$\Delta \epsilon \, C_m = \frac{w_{total}}{V} - \frac{w_W}{V_0} = \frac{1}{V/V_0} \left(\frac{w_{total}}{V_0} \right) - \frac{w_W}{V_0} \qquad (12)$$

The average upwash velocity w_W at the wing is also determined from the power-on lift coefficient. In reference 7 it is suggested that the power-off lift coefficient be used to determine the upwash velocity at the wing because, for most single- or twin-engine airplanes, the increase in the lift due to the slipstream is primarily due to an increase in local velocity over the wings rather than to a marked increase in the circulation of the wing, although the circulation distribution is altered by the slipstream. Thus, for practical purposes, the same system of simple horseshoe vortices could be used to represent the wing with or without a slipstream, and the lift coefficient of the wing measured without power could be used to compute the boundary-induced upwash velocity at the wing due to the vortices. The jet effect of the inclined slipstream was neglected in the arguments of reference 7. A few approximate calculations based on equation (8) having indicated that the effect of the inclined slipstream may be roughly approximated by using the power-on lift coefficient instead of the power-off lift coefficient, this procedure will be used in this paper.

The correction to the measured pitching moments of a powered model depends upon the power-on lift, the correction factors, the slipstream velocity at the tail plane, and the stabilizer effectiveness. The correction is determined in the form

$$\Delta C_m = \Delta \epsilon \, C_m \left(\frac{\partial C_m}{\partial i_t} \right) 57.3 \qquad (13)$$

where $\Delta \epsilon \, C_m$ is given in equation (12) and $\partial C_m / \partial i_t$ is the measured stabilizer effectiveness for the given conditions.

In general, both $\Delta \epsilon \, C_m$ and $\partial C_m / \partial i_t$ are functions of C_L or angle of attack α and both may be computed; it is, however, more accurate and usually simpler to determine $\partial C_m / \partial i_t$ experimentally. If air-flow surveys

are not available, the average velocity ratio may be determined, to a first approximation, by use of $\partial C_m/\partial i_t$ as a function of C_L as follows:

$$\left(\frac{V_s}{V_o}\right)_{av} = \sqrt{\left(\frac{q_s}{q_o}\right)_{av}} = \sqrt{\frac{(\partial C_m/\partial i_t)_{power\ on}}{(\partial C_m/\partial i_t)_{free\ stream}}} = \left(\frac{V}{V_o}\right)_{av} \quad (14)$$

If the tail does not lie in the wing or fuselage wake, the power-off stabilizer effectiveness may be used for the free-stream stabilizer effectiveness. It may be necessary to plot the power-off stabilizer effectiveness as a function of lift coefficient and to extrapolate to about zero lift, or even to some negative lift where the tail may be considered out of the wake.

When equation (12) is converted to correction-factor form, the angular pitching-moment correction in radians is

$$\Delta \epsilon C_m = \left[\frac{1}{\sqrt{(q/q_o)_{av}}} (\delta_{total} C_{L_p})_{w+f} - (\delta_w C_{L_p})_{w+f}\right] \frac{S}{C} \quad (15)$$

and equation (13) becomes

$$\Delta C_m = \left[\frac{1}{\sqrt{(q/q_o)_{av}}}(\delta_{total} C_{L_p})_{w+f} - (\delta_w C_{L_p})_{w+f}\right] \frac{S}{C}\left(\frac{\partial C_m}{\partial i_t}\right) 57.3 \quad (16)$$

The added correction due to wake or slipstream displacement is not included in this formula and must be separately determined.

The values of the correction factors are for the effective span and the effective height of the lifting line above the tunnel center line. The value of x to be used in determining δ_{total} is equal to the distance between the quarter-chord point of the wing and the three-quarter-chord point of the tail, because the three-quarter-chord point is the best measure of the effective angle of attack of the tail (reference 8).

All factors in equation (16) should be determined as functions of the tunnel angle of attack; and the final correction will be a function of the tunnel angle of attack. A simple straight-line expression for the correction may usually be determined in the form of

$$\Delta C_m = K + K_1 \alpha$$

where the values of K and K_1 depend upon the particular power and model condition. For closed-type wind tunnels the pitching-moment correction is to be added to the experimentally determined values of C_m.

Elevator-free angle correction.- The measured free-floating angle of the elevator for elevator-free tests will be in error due to the jet-boundary effect. The correction is determined in a manner similar to that for the pitching-moment correction. In fact, the correction angle $\Delta \epsilon_{C_m}$ is used to determine the elevator-free angle correction. It is necessary to make elevator-free tests at two stabilizer angles and thus to determine $\partial \delta_e / \partial i_t$ (and $\partial C_m / \partial i_t$) as functions of C_L.

Then the correction to the measured elevator-free angle will be

$$\Delta \delta_e = \Delta \epsilon_{C_m} \left(\frac{\partial \delta_e}{\partial i_t} \right) 57.3 \tag{17}$$

and to the measured pitching moment

$$\Delta C_m = \Delta \epsilon_{C_m} \left(\frac{\partial C_m}{\partial i_t} \right) 57.3 \tag{18}$$

where $\partial C_m / \partial i_t$ is for the elevator-free conditions.

Elevator hinge-moment correction.- The measured elevator hinge moments will also be slightly in error because of jet-boundary effect. The effect is usually quite small and often within the experimental accuracy of the measuring equipment. The general method of attack is similar

to the method used to determine the pitching-moment correction. The elevator hinge-moment correction

$$\Delta C_{h_e} = \Delta \epsilon C_m \left(\frac{\partial C_{h_e}}{\partial i_t}\right) 57.3 \tag{19}$$

where $\partial C_{h_e}/\partial i_t$ may be determined experimentally or calculated approximately as desired.

Streamline-curvature correction.- An estimate of the streamline-curvature correction to the lift of the wings was made by use of the downwash correction factors and is given in appendix A.

NUMERICAL VALUES OF δ

FOR 7- BY 10-FOOT CLOSED WIND TUNNEL

The numerical values of the jet-boundary corrections will be given in the form of the usual correction factors. (See equations (3) and (4).) In order to calculate the correction factors it is necessary to determine the average upwash velocity at the wings, the upwash velocity at the center section of the wing, and the additional upwash velocity at the tail plane $w_{a_{c.s}}$, due to the influence of the jet boundaries upon the wing.

The average upwash velocity w_w is the value of upwash velocity generally used to determine the corrections for induced angle of attack and induced drag. Methods of computation are adequately described in reference 3. This average upwash velocity should be calculated for various values of vortex semispan and for several off-center positions of the lifting line in the tunnel. Numerical values of the correction factor δ_w are given in figure 3 for the 7- by 10-foot closed wind tunnel. As previously mentioned, the accuracy is increased by the use of an effective span of about 0.9 the actual span for wings and an effective span equal to the actual flap span for partial-span flaps.

The upwash velocity at the center section of the wing $w_{c.s.}$ is calculated from the first terms of equation

(1) or by the following equation in which na is substituted for y, $mh + (-1)^m d$ for z, and the necessary summations are indicated:

$$w_{c.s.} = \frac{\Gamma}{4\pi} \sum^n \sum^m \left\{ (-1)^m \left[\frac{na - s}{(na - s)^2 + (mh + (-1)^m d)^2} \right.\right.$$

$$\left.\left. - \frac{na + s}{(na + s)^2 + (mh + (-1)^m d)^2} \right] \right\} \quad (20)$$

The integers n and m take the positive and negative values that define the important image tunnels.

The center-section upwash velocity should also be calculated for various vortex semispans and several off-center lifting-line positions. Numerical values of the correction factor $\delta_{c.s.}$ are given in figure 3 for the -7- by 10-foot closed wind tunnel.

The increase in the upwash velocity at the tail plane over the upwash velocity at the center section of the wing $w_{a_{c.s.}}$ must be calculated. The last terms of equation (1) or the following equations, for which the substitutions of na for y and $mh + (-1)^m d - d_t$ for z have already been made, may be used for the calculations. The values n and m are for the important image tunnels.

$$w_{a_{c.s.}} = \sum^n \sum^m \frac{(-1)^m \Gamma}{4\pi} \left(\left\{ \left[\frac{x(na - s)}{\sqrt{(na-s)^2 + x^2 + (mh+(-1)^m d - d_t)^2}} \right] \right.\right.$$

$$\left. \times \left[\frac{1}{x^2 + (mh+(-1)^m d - d_t)^2} + \frac{1}{(na-s)^2 + (mh+(-1)^m d - d_t)^2} \right] \right\}$$

$$- \left\{ \left[\frac{x(na + s)}{\sqrt{(na + s)^2 + x^2 + (mh + (-1)^m d - d_t)^2}} \right] \right.$$

$$\left.\left. \times \left[\frac{1}{x^2 + (mh+(-1)^m d - d_t)^2} + \frac{1}{(na+s)^2 + (mh+(-1)^m d - d_t)^2} \right] \right\} \right) \quad (21)$$

where d_t is the height of the tail above the tunnel
center line. The numerical values of upwash velocity are
the same for negative as for positive values of n. The
calculations need not therefore be repeated for both pos-
itive and negative values of n. Values of additional
upwash $w_{a_{c.s.}}$ for various vortex spans and tail lengths
for several off-center positions of the lifting line and
the tail should be calculated. Numerical values of the
correction factor $\delta_{a_{c.s.}}$ for the 7- by 10-foot closed
wind tunnel are given in figure 4.

The total upwash correction factor

$$\delta_{total} = \delta_{c.s.} + \delta_{a_{c,s.}}$$

is given in figure 5 and is the sum of $\delta_{c.s.}$ from fig-
ure 3 and $\delta_{a_{c.s.}}$ from figure 4.

METHOD OF APPLYING CORRECTIONS

The step-by-step procedure for determining the cor-
rections to the downwash angle, the wake and slipstream
location, the pitching moment, the elevator free-floating
angle, and the elevator hinge moment is as follows:

(The curves presented in the figures referred to ap-
ply only to a 7- by 10-foot closed wind tunnel.)

1. The boundary-correction factor δ_w is determined
from figure 3, the value of effective semispan s being
taken as 0.9 the wing semispan for the wing correction
and equal to the flap semispan for the flap correction.
The value of d is equal to d_g , the vertical height of
the tip vortices (trailing edge of the 0.9 semispan sta-
tion).

2. The effective height of the lifting line above
the tunnel center line is obtained from equation (7) in
which the value of d_g was determined, in step 1, as a
function of angle of attack. The value of δ_{total} is
then determined from figure 5 for the proper values of
d_t , d, and x. As the value of δ_{total} varies with

model attitude, δ_{total} must be determined as a function of model angle of attack. In case of a flapped wing, a similar calculation to determine δ_{total} due to the flaps must also be made.

3. From power-off and power-on curves of $\partial C_m/\partial i_t$ as a function of lift coefficient, the value of $(q/q_0)_{av}$ may be determined from equation (14). If dynamic-pressure surveys are to be made, the value of q/q_0 will be available from the surveys.

4. The correction to the downwash angle is found by substituting into equation (10) the value of q/q_0; the wing and tunnel areas, S and C; and δ_{total} as found in steps 2 and 3. This downwash-angle correction is in degrees and is added to the measured downwash angle; that is, the downwash angle measured in a closed wind tunnel is lower than it would be in free air.

5. The correction to the wake center-line location (or the slipstream center-line location) $\Delta z'$ at any point x behind the model is found from equation (11) where the value of δ_{total} as a function of x is obtained from figure 5. The value of d_t to be used in determining δ_{total} is equal to the distance of the wake or slipstream center line above the center line of the tunnel. The wake correction may be applied as a constant vertical displacement of all points outside the slipstream having a horizontal ordinate equal to x. The slipstream correction may be applied to all points inside the slipstream. The corrections to the wake location and to the slipstream location will, in general, be different. (Two corrections to the angle of downwash have been calculated. One of the corrections changes the angle of downwash at each point and the other correction effectively changes the vertical location of that point with respect to the airplane. The displacement correction is sometimes more important than the downwash-angle correction for some powered models.)

6. The correction to the pitching moment ΔC_m is found by substituting the values of $\sqrt{(q/q_0)_{av}}$, C_{L_p}, δ_{total}, δ_w, and $\partial C_m/\partial i_t$ into equation (16). The values

of these factors are experimentally determined (or estimated if necessary) as functions of the tunnel angle of attack. Values for δ_w and δ_{total} were determined in steps 1 and 2 and for $\sqrt{(q/q_0)_{av}}$ in step 3. The final correction as obtained from equation (16) will be a function of tunnel angle of attack. The correction is to be added to the measured pitching moments for closed-type wind tunnels.

7. An additional correction to the pitching moment must be made for the wake or the slipstream displacement calculated in step (5); that is, an additional correction angle $\Delta\epsilon_z$, must be calculated. This correction corresponding to the change in vertical location of the tail $\Delta z'$ with respect to the wake is calculated from the charts in reference 5 or from downwash surveys of the same or of similar powered models.

8. The corrections for the measured elevator free-floating angle $\Delta\delta_e$ and the pitching moment ΔC_m are given by equations (17) and (18) with the value of $\Delta\epsilon_{C_m}$ as calculated in step 6. (The value of $\Delta\epsilon_z$, as calculated in step 7, may be added to $\Delta\epsilon_{C_m}$.)

9. The correction to elevator hinge-moment coefficient ΔC_{h_e} is given in equation (19). The values of $\Delta\epsilon_{C_m}$ and $\Delta\epsilon_z$, are the same as that used in step 8.

CONCLUDING REMARKS

The methods presented may be used to determine the jet-boundary corrections to the downwash angle, the wake and slipstream location, the pitching moment, the elevator free-floating angle, and the elevator hinge moment for powered models tested in rectangular wind tunnels. Numerical values of the various correction factors were presented for 7- by 10-foot closed wind tunnels. The methods were presented and discussed in some detail. The direct effect of the slipstream and the secondary effects of the jet-boundary induced wake or slipstream displacement upon the measured downwash and the measured pitching moments have been shown to be important and should not be neglected.

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APPENDIX A

STREAMLINE-CURVATURE CORRECTION TO THE LIFT OF WINGS

Inasmuch as the induced upwash velocity varies with the distance from the lifting line, there is an effective curvature of the streamlines. This curvature has the same resultant effect as a change in camber of the airfoil. In a closed-type wind tunnel the streamlines curve upward, thereby effectively increasing the positive camber of the airfoil; thus, the airfoil has a higher lift in the tunnel than it would have in free air. The correction may be applied as a lift correction at the given angle of attack or it may be applied in the form of an increased angle-of-attack correction. In fact, thin-wing-section theory indicates that, if the streamlines passing over the wing chord are arcs of circles (as they are to a first approximation), exactly half the correction should be applied as a lift correction and half as an increased angle-of-attack correction. The correction will be determined in each of the three ways: first, as increased angle-of-attack correction (with no correction to the lift); then as a lift correction (with the usual wing angle-of-attack correction); and, finally, following wing-section theory, with half the correction as a lift correction and half as an increased angle-of-attack correction.

The change in the effective angle of attack of the airfoil due to a change in streamline curvature (circular camber) is approximately equal to the change in the angle at the three-quarter-chord point of the wing (reference 8). If R is the radius of curvature of the streamlines, the change in angle (correction angle) in radians is

$$\Delta \alpha_{\frac{3c}{4}} = \frac{1}{2} \frac{c}{R}$$
(22)

The radius of curvature is found as follows:

To a first approximation

$$R = V \frac{1}{\frac{\partial w}{\partial x}}$$
(23)

$$\frac{w}{V} = \delta_a \frac{S}{C} C_L \qquad (24)$$

and

$$\frac{1}{R} = \frac{S}{C} C_L \frac{\partial \delta_a}{\partial x} \qquad (25)$$

The slope $\partial \delta_{a_{c.s.}}/\partial x$ may be determined from cross plots ($\delta_{a_{c.s.}} = f(x)$) from figure 4. An approximate average value for the 7- by 10-foot closed tunnel is $\frac{\partial \delta_{a_{c.s.}}}{\partial x} = 0.033$. This same slope will be assumed constant for all points along the wing span.

The correction to the angle of attack (due to streamline curvature) then becomes the change in induced upwash between the one-quarter and the three-quarter chord points. The value of the angle in radians is

$$\Delta \alpha_{\frac{3c}{4}} = \frac{1}{2}c \frac{S}{C} C_L (0.033) \qquad (26)$$

The total correction to the angle of attack, in radians, is

$$\Delta \alpha_{total} = \delta_w \frac{S}{C} C_L + \Delta \alpha_{\frac{3c}{4}}$$

or, in degrees,

$$\Delta \alpha_{total} = (\delta_w + 0.017c) \frac{S}{C} C_L (57.3) \qquad (27)$$

If it is desired to apply the streamline-curvature correction as a lift correction, the angle-of-attack correction in degrees will be, as usual,

$$\Delta \alpha = \delta_w \frac{S}{C} (C_L)(57.3) \qquad (28)$$

and the lift-coefficient correction is approximately equal to

$$\Delta C_L = -\Delta \alpha_{\frac{3c}{4}} \frac{dC_L}{d\alpha} \tag{29}$$

If $dC_L/d\alpha = 0.07$

$$\Delta C_L = -\frac{1}{2}(0.033)(0.07)(57.3)c \frac{S}{C} C_L$$

or

$$\Delta C_L = -0.066c \frac{S}{C} C_L \tag{30}$$

From equations (27) and (30) the corrections to be applied, if both the angle of attack and the lift coefficient are to be corrected, are

$$\Delta \alpha_{total} = (\delta_w + 0.008c) \frac{S}{C} C_L (57.3) \tag{31}$$

and

$$\Delta C_L = -0.033c \frac{S}{C} C_L$$

The numerical values given, of course, apply only to 7- by 10-foot closed wind tunnels. The corrections are to be added to the tunnel values. The lift-coefficient correction is about 1 to 2 percent of the lift coefficient for models of the size usually tested in a 7- by 10-foot wind tunnel.

APPENDIX B

C_L	lift coefficient
ΔC_L	correction to the lift coefficient
α	angle of attack
$\Delta \alpha$	correction to angle of attack
ϵ	angle of inclination of downwash or slip-stream
$\Delta \epsilon$	correction to downwash angle $\left(\delta \frac{S}{C} C_L \right)$
δ	jet-boundary correction factor $\left(\frac{C}{4s} \frac{w}{\Gamma} \right)$
a	tunnel breadth
h	tunnel height
C	tunnel area (ah)
m	integer defining number of images in Z direction
n	integer defining number of images in Y direction
S	wing area
s	semispan of simple horseshoe vortex
b	wing span
c	chord
d	effective height of vortex system above tunnel center line
d_t	height of point in question (tail) above tunnel center line
Γ	circulation strength of horseshoe vortex
V	velocity parallel to X axis

q   dynamic pressure.

w   induced vertical velocity parallel to Z axis

x   distance from lifting line to point in question, parallel to X axis

y   distance from center section of lifting line, parallel to Y axis

z   distance from lifting line, parallel to Z axis

z' vertical displacement of wake or slipstream

$\Delta z'$ correction to vertical displacement of wake or slipstream

$\left(\delta C_{L_p}\right)_w$ product of correction factor due to wing and increment of lift coefficient due to wing

$\left(\delta C_{L_p}\right)_f$ product of correction factor due to flap and increment of lift coefficient due to flap

$$\left(\delta C_{L_p}\right)_{w+f} = \left(\delta C_{L_p}\right)_w + \left(\delta C_{L_p}\right)_f$$

$\partial C_m / \partial i_t$ change in pitching moment per degree change in stabilizer angle (stabilizer effectiveness)

$\partial \delta_e / \partial i_t$ change in elevator free-floating angle per degree change in stabilizer angle

$\partial C_{h_e} / \partial i_t$ change in elevator hinge-moment coefficient per degree change in stabilizer angle

ΔC_m correction to pitching-moment coefficient

$\Delta \delta_e$ correction to elevator free-floating angle

ΔC_{h_e} correction to elevator hinge-moment coefficient

R	radius of curvature of streamlines
r, θ	position of point with respect to center line of a two-dimensional slipstream

Subscripts:

a	additional
$\frac{3c}{4}$	at three-quarter chord point
av	average
s	slipstream
o	free stream
c.s.	at center section of wing
C_m	pitching moment
d	downwash
t	tail
total	total
w	average over wing
p	with power
T.E.	trailing edge of wing
z'	due to vertical displacement
g	geometric position of tip vortices

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TABLE I

VALUES OF ADDITIONAL BOUNDARY-CORRECTION FACTOR $\delta a_{c.s.}$ FOR EACH IMAGE TUNNEL

Values of $\delta a_{c.s.}$ at a point 3 feet behind the lifting line of a 3-foot semispan horseshoe vortex located on the center line of the 7- by 10-foot wind tunnel for positive or negative values of m and n less than 15. If both positive and negative values of m and n are considered: $\sum \delta a_{c.s.} = 0.07894$ for all image tunnels; $\sum \delta a_{c.s.} = 0.07948$ for important image tunnels only; $\delta a_{c.s.}$ is positive, though less than $|0.00001|$ in 398 image tunnels (indicated as +0); $\delta a_{c.s.}$ is negative, though less than $|0.00001|$ in 384 image tunnels (indicated as -0).

| m \ n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0		[a]0.00957	[a]0.00111	[a]0.00029	[a]0.00012	0.00006	0.00006	+0	+0	+0	+0	+0	+0	+0	+0	+0
1	[a]0.03517	[a].00035	[a]-.00058	[a]-.00023	[a]-.00012	-.00006	-.00006	-0	-0	-0	-0	-0	-0	-0	-0	-0
2	[a]-.00560	[a]-.00163	[a]-0	[a].00012	[a].00006	.00006	.00006	+0	+0	+0	+0	+0	+0	+0	+0	+0
3	[a].00175	[a].00093	[a].00017	-0	-.00006	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0
4	[a]-.00076	[a]-.00052	[a]-.00017	-.00006	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0
5	[a].00041	[a].00029	[a]-.00017	.00006	+0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0
6	-.00023	-.00017	-.00012	-.00006	-0	-0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0
7	.00012	.00012	.00012	.00006	+0	+0	+0	-0	-0	-0	-0	-0	-0	-0	-0	-0
8	-.00012	-.00006	-.00006	-.00006	-0	-0	-0	-0	+0	+0	+0	+0	+0	+0	+0	+0
9	.00006	.00006	.00006	.00006	+0	+0	+0	+0	+0	-0	-0	-0	-0	-0	-0	-0
10	-.00006	-.00006	-.00006	-0	-0	-0	-0	-0	-0	-0	+0	+0	+0	+0	+0	+0
11	.00006	.00006	.00006	+0	+0	+0	+0	+0	+0	+0	+0	+0	-0	-0	-0	-0
12	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	+0	+0	+0	+0
13	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	-0	-0	-0
14	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	-0	+0
15	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	-0

[a]Important image tunnels.
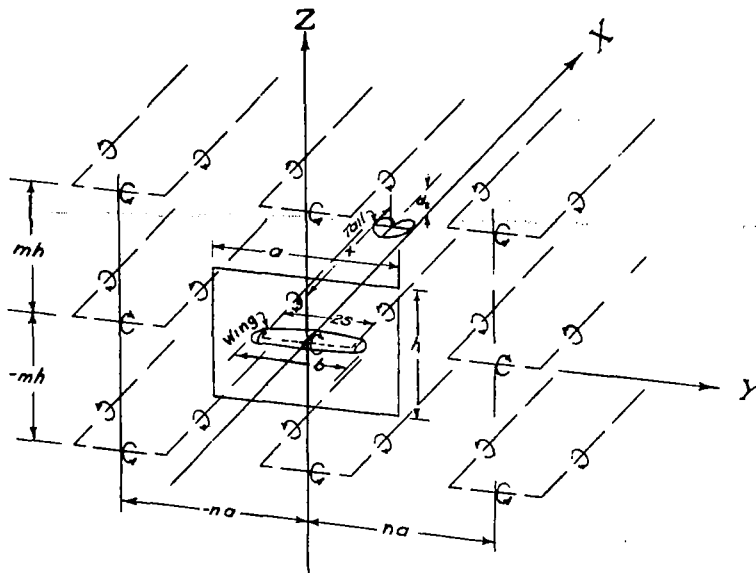
Figure 1.— Three-dimensional arrangement of doubly infinite image pattern to satisfy the boundary conditions for a wing on the center line of a closed rectangular tunnel.
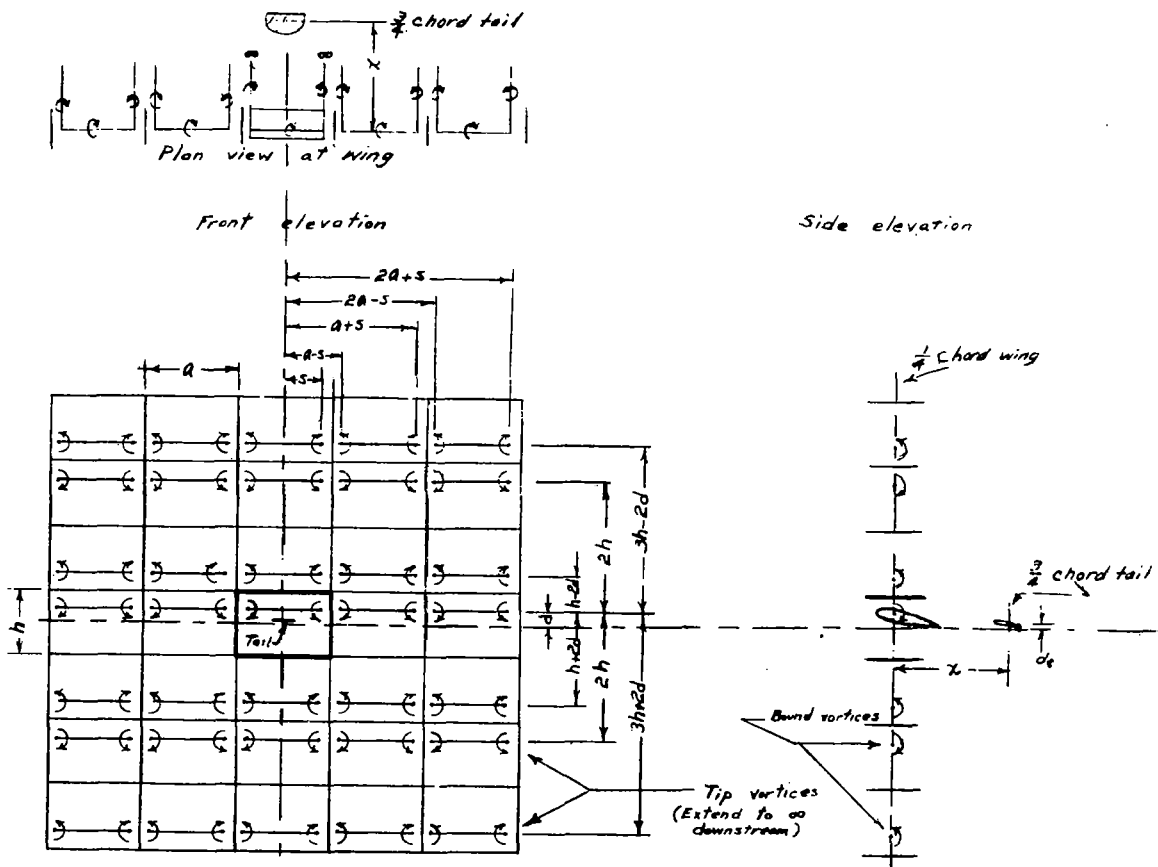

FIGURE 2.— Arrangement of the doubly infinite image pattern to satisfy the boundary conditions for off-center wing locations in a closed rectangular tunnel.
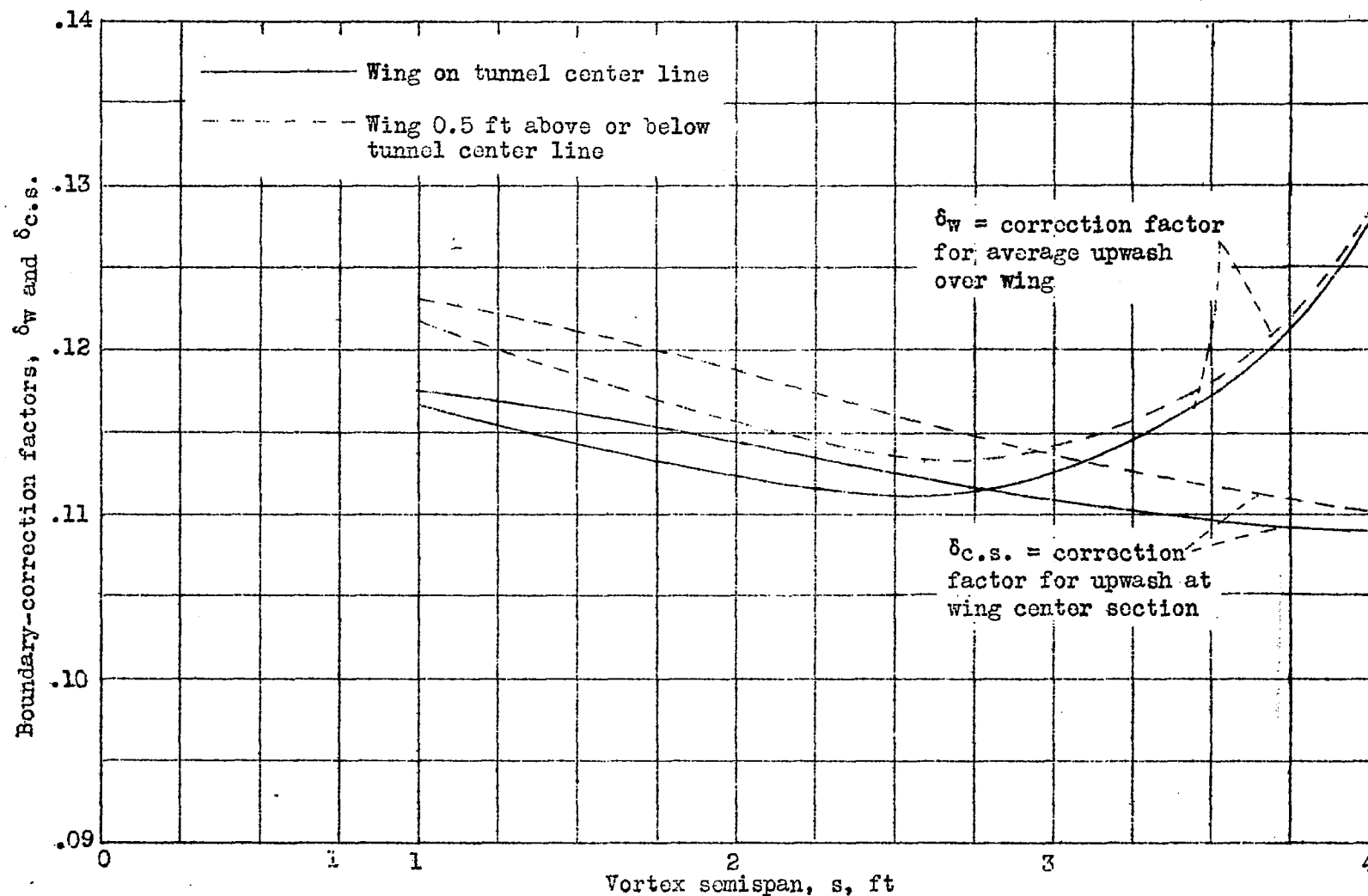
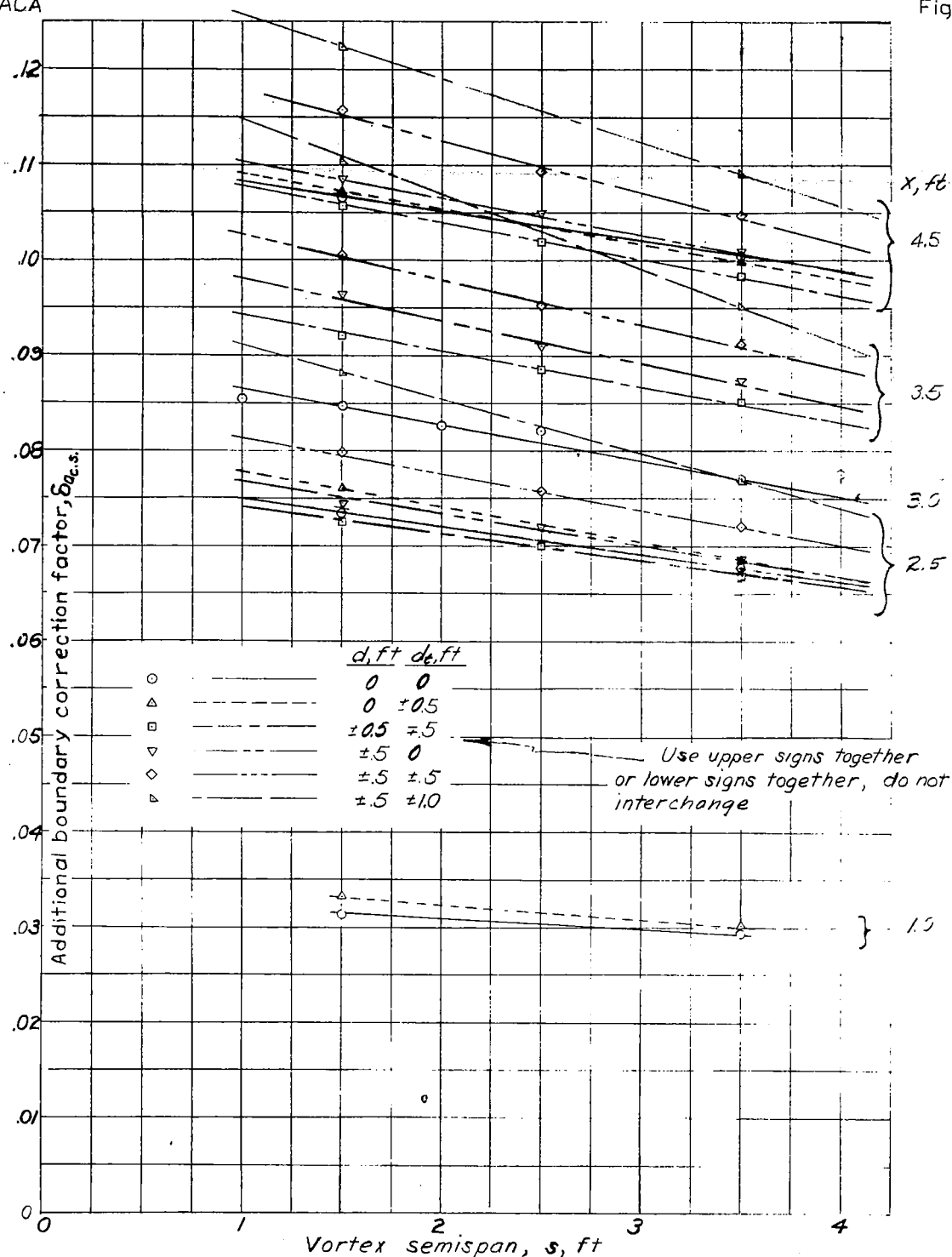Figure 3.— Boundary correction factors at the lifting line for 7-by 10-foot closed rectangular wind tunnels.

Fig. 3

Fig. 4

FIGURE 4.- The additional boundary-correction factor for 7-by 10-foot closed wind tunnels. (The boundary correction factor behind the wing in addition to that at the center section of the wing.)
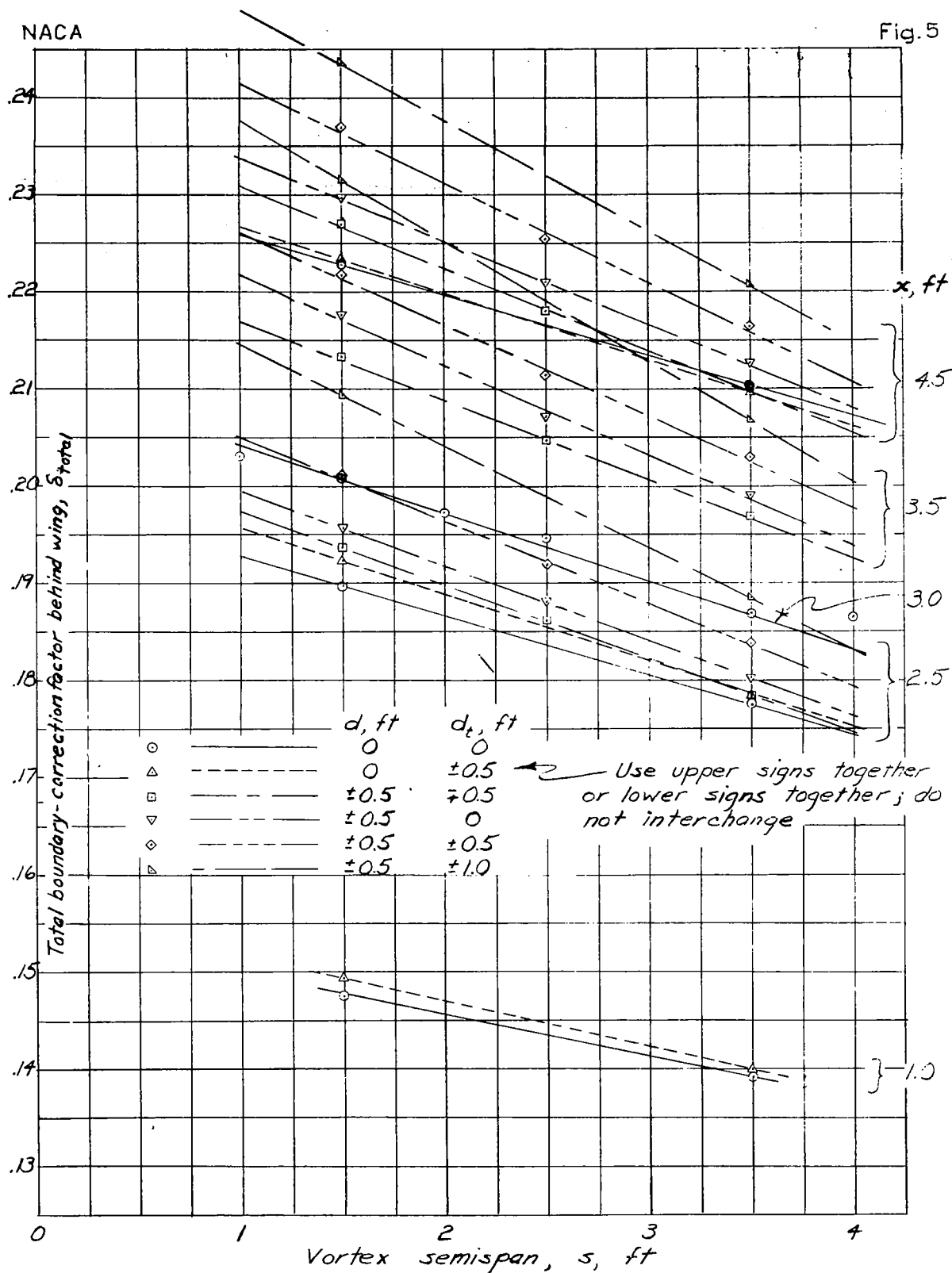
FIGURE 5. — Total boundary-correction factor behind wing for 7-by-10-foot closed rectangular wind tunnels.